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Electric Dipole Moments of Dyon and 'Electron'

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Abstract

The electric and magnetic dipole moments of dyon fermions are calculated within $N = 2$ supersymmetric Yang-Mills theory including the θ-term. It is found, in particular, that the gyroelectric ratio deviates from the canonical value of 2 for the monopole fermion (n_{m}=1, n_{e}=0) in the case $\theta \neq 0$. Then, applying the S-duality transformation to the result for the dyon fermions, we obtain an explicit prediction for the electric dipole moment (EDM) of the charged fermion ('electron'). It is thus seen that the approach presented here provides a novel method for computing the EDM induced by the θ-term.

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§1. Introduction

The θ-term in gauge theory violates the CP symmetry, and hence it can generally induce an electric dipole moment (EDM) for a particle with spin. However, at present, there exists no general, established method to calculate this EDM. The reason for the difficulty is that the θ-term is a total derivative and therefore it neither changes the equation of motion nor contributes to the Feynman rules. Thus no calculation in which the θ-term itself is treated perturbatively can provide a method for determining the EDM. In the conventional calculation of the EDM of the neutron in QCD, this problem is avoided by first performing a chiral transformation to replace the θ-term with the complex phase of the quark mass term through the chiral $U(1)$ anomaly. Then, because such a mass term with a complex phase is readily treated in a perturbative calculation, the EDM can be computed. In this computation, the anomaly gives a non-perturbative information.

Here we propose an interesting and novel method to calculate the EDM of a charged fermion. We consider $N = 2$ $SU(2)$ supersymmetric Yang-Mills theory, in which there exists a BPS monopole multiplet consisting of a monopole scalar and a monopole fermion ($J = 1/2$). We apply the S-duality transformation to this system, and as a result, the monopole multiplet turns to a charged BPS multiplet consisting of a scalar and a spin 1/2 fermion, which we call an 'electron', for brevity. Note that the two systems before and after the S-duality transformation are distinct because their vacua differ. To distinguish these systems, we refer to them as the 'monopole world' and the 'electron world', respectively.

We compute the EDM of the fermion as follows. First, in the original (i.e. monopole) world, we calculate the electromagnetic fields around a monopole fermion placed at the origin. When the θ-term is added, the monopole acquires an electric charge (through Witten effect[1]), in addition to the magnetic charge, and thus it becomes a dyon. We use the Julia-Zee classical dyon solution corresponding to its electric and magnetic charges. The electromagnetic fields around the monopole fermion can be found by applying the supersymmetry (SUSY) transformation to the classical Julia-Zee dyon solution. As discussed in previous works,[2),3)] the first-order term in the SUSY transformation parameter gives a fermion zero-mode solution representing the fermion partner of the monopole. The second-order term gives the desired electromagnetic field around the monopole fermion. We can determine the magnetic dipole moment and the electric dipole moment carried by the monopole fermion directly from the asymptotic form of this electromagnetic field.

Next we apply the S-duality transformation to the monopole system. The BPS monopole multiplet is thereby transformed to the BPS electron multiplet in the electron world. We then obtain the electromagnetic field around the electron by applying the S-duality transformation

to that around the monopole fermion. The EDM, μ_{e}, and magnetic (dipole) moment, μ_{m}, can be determined from the asymptotic form of this electromagnetic field. Our result is the following:

$$\mu_{\mathrm{m}} = \frac{e}{m}J, \qquad \mu_{\mathrm{e}} = \frac{e^2\theta}{8\pi^2}\frac{e}{m}J, \tag{1}$$

with $J = 1/2$ standing for the spin. The magnetic (dipole) moment $\mu_{\mathrm{m}} = 2 \times (e/2m)J$ is identically the Dirac value, corresponding to the gyromagnetic ratio $g_{\mathrm{m}} = 2$. By contrast, the EDM is given by the 'Dirac value', eJ/m, multiplied by a factor of $e^2\theta/8\pi^2$.

This paper is organized as follows. In §2, we compute the electromagnetic fields around a dyon fermion by performing a finite supersymmetry transformation of the Julia-Zee dyon solution, and we thus obtain both the EDM and the magnetic moment of the dyon fermion. In §3, we elucidate the $SL(2;\mathbb{Z})$ transformation of the electromagnetic field strength. Then, we carry out the S-duality transformation, converting the monopole fermion state into an electron state, and find the EDM of the electron. The final section is devoted to discussion of the results.

§2. Electric dipole moment of a dyon

We consider $N = 2$ $SU(2)$ supersymmetric Yang-Mills theory with the Lagrangian

$$\begin{aligned}\mathcal{L} = \mathrm{Tr}\biggl(&-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}D^\mu S D_\mu S + \frac{1}{2}D^\mu P D_\mu P + \frac{e^2}{2}[S,P]^2 \\ &+i\bar{\psi}\gamma^\mu D_\mu\psi - e\bar{\psi}[S,\psi] + ie\bar{\psi}\gamma_5[P,\psi] + \frac{\theta e^2}{32\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}\biggr),\end{aligned} \tag{2}$$

where all fields are written in matrix form; i.e., here we have $S = S^a T^a$, etc., with the generators T^a in the fundamental representation satisfying the relation $\mathrm{tr}\, T^a T^b = (1/2)\delta^{ab}$. We define the operator Tr as $\mathrm{Tr} \equiv 2\,\mathrm{tr}$, so that $\mathrm{Tr}\, T^a T^b = \delta^{ab}$. This Lagrangian is invariant, up to total derivative terms, under the $N = 2$ supersymmetry transformations

$$\begin{aligned}\delta A_\mu &= i\bar{\tilde{\alpha}}\gamma_\mu\psi - i\bar{\psi}\gamma_\mu\tilde{\alpha}, \\ \delta S &= i\bar{\tilde{\alpha}}\psi - i\bar{\psi}\tilde{\alpha}, \\ \delta P &= \bar{\tilde{\alpha}}\gamma_5\psi - \bar{\psi}\gamma_5\tilde{\alpha}, \\ \delta\psi &= \left(\frac{1}{2}\gamma^{\mu\nu}F_{\mu\nu} - \gamma^\mu D_\mu S + i\gamma^\mu D_\mu P\gamma_5 - e[P,S]\gamma_5\right)\tilde{\alpha},\end{aligned} \tag{3}$$

where $\gamma^{\mu\nu} = \gamma^{[\mu}\gamma^{\nu]} \equiv (1/2)[\gamma^\mu, \gamma^\nu]$, and $\tilde{\alpha}$ is a Grassmann-valued Dirac spinor.*) Witten showed in Ref. 1) that the θ-term induces an electric charge of $+e\theta/2\pi$ for a magnetic

*) We employ the convention in which $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\eta^{\mu\nu}$, $\eta^{\mu\nu} = \mathrm{diag}(+1,-1,-1,-1)$, and $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$.

monopole with unit magnetic charge, $g = 4\pi/e$. *) thereby implying the existence of dyons. In accordance with this fact, a classical dyon solution exists that has long been known as the Julia-Zee dyon:[7]

$$\begin{aligned} A_0^a &= \frac{\hat{r}^a}{er} H(evr)\sinh\gamma, \qquad A_i^a = \epsilon^a{}_{ij}\hat{r}^j \frac{1-K(evr)}{er}, \\ S^a &= \frac{\hat{r}^a}{er} H(evr)\cosh\gamma, \qquad \psi^a = P^a = 0, \end{aligned} \tag{4}$$

where the functions $K(x)$ and $H(x)$ are given by

$$K(x) = \frac{x}{\sinh x}, \tag{5}$$

$$H(x) = x\coth x - 1. \tag{6}$$

The electric and magnetic charges, Q_e and Q_m, of the unbroken $U(1)$ gauge field $F_{\mu\nu} \equiv S^a \cdot F_{\mu\nu}^a/a$, with $a \equiv \sqrt{S^a S^a}$ at the spatial infinity, are defined by

$$\begin{aligned} Q_\mathrm{e} &\equiv \int_{S_\infty} \boldsymbol{E}\cdot d\boldsymbol{S} = \frac{1}{a}\int d^3x\, \boldsymbol{E}^a \cdot (\boldsymbol{D}S)^a, \\ Q_\mathrm{m} &\equiv \int_{S_\infty} \boldsymbol{B}\cdot d\boldsymbol{S} = \frac{1}{a}\int d^3x\, \boldsymbol{B}^a \cdot (\boldsymbol{D}S)^a. \end{aligned} \tag{7}$$

Here we have defined the quantities

$$\boldsymbol{E}^a \equiv (F_{01}^a,\, F_{02}^a,\, F_{03}^a), \qquad \boldsymbol{B}^a \equiv (-F_{23}^a,\, -F_{31}^a,\, -F_{12}^a). \tag{8}$$

Inserting the solution (4) into the definition of a and Eq. (7), we find that $a = v\cosh\gamma$ and the Julia-Zee dyon carries

$$Q_\mathrm{m} = \frac{4\pi}{e} \equiv g, \qquad Q_\mathrm{e} = -\frac{4\pi}{e}\sinh\gamma. \tag{9}$$

Here, g is the unit magnetic charge of monopole. Classically, the parameter γ is an arbitrary real constant. However, as argued by Witten, the generator N of the unbroken $U(1)$ gauge transformation $\delta A_\mu^a = D_\mu S^a/ea$, which is found using the Noether method to be

$$N = \frac{\partial\mathcal{L}}{\partial(\partial_0 A_\mu^a)}\delta A_\mu^a = \frac{Q_\mathrm{e}}{e} - \frac{\theta e Q_\mathrm{m}}{8\pi^2}, \tag{10}$$

should have integer eigenvalues n_e. Using the relation $eQ_\mathrm{m} = 4\pi$ in (9) for the magnetic charge of the Julia-Zee dyon, we see that the parameter γ, or equivalently, the electric charge Q_e, is quantized as follows in the case that the Lagrangian includes the CP-violating θ-term:

$$Q_\mathrm{e} = -\frac{4\pi}{e}\sinh\gamma = n_\mathrm{e} e + \frac{e\theta}{2\pi}. \tag{11}$$

*) We have chosen the sign of the θ-term in the Lagrangian (2) such that the Witten charge is $+\theta e/2\pi$. This is the same as the convention of Alvarez-Gaumé and Hassan,[4] but it is opposite those of Seiberg and Witten[5] and Harvey.[6]

The dyon mass is given by

$$
\begin{aligned}
m &= \int d^3x\,\frac{1}{2}\Big[(\boldsymbol{E}^a)^2+(\boldsymbol{B}^a)^2+(\boldsymbol{D}S^a)^2+(D_0S^a)^2\Big]\\
&= \int d^3x\,\frac{1}{2}\Big[(\boldsymbol{E}^a-\boldsymbol{D}S^a\sin\alpha)^2+(\boldsymbol{B}^a-\boldsymbol{D}S^a\cos\alpha)^2+(D_0S^a)^2\Big]\\
&\qquad\qquad +a(Q_{\rm e}\sin\alpha+Q_{\rm m}\cos\alpha),
\end{aligned}
\tag{12}
$$

and thus it satisfies the relation $m \geq a(Q_{\rm e}\sin\alpha+Q_{\rm m}\cos\alpha)$ for any α. Because the quantity on the right-hand side is maximal when $\tan\alpha = Q_{\rm m}/Q_{\rm e}$, we obtain a bound, called Bogomol'nyi bound, expressed by the following:

$$m \geq a\sqrt{Q_{\rm m}^2+Q_{\rm e}^2}. \tag{13}$$

The Julia-Zee dyon is characterized as the BPS state for which the bound is saturated, and hence it satisfies the first-order Bogomol'nyi equations

$$\boldsymbol{B}^a = \boldsymbol{D}S^a\cos\alpha, \qquad \boldsymbol{E}^a = \boldsymbol{D}S^a\sin\alpha, \qquad D_0S^a = 0, \tag{14}$$

with $\cos\alpha = Q_{\rm m}/\sqrt{Q_{\rm m}^2+Q_{\rm e}^2}$ and $\sin\alpha = Q_{\rm e}/\sqrt{Q_{\rm m}^2+Q_{\rm e}^2}$. Substituting the expressions (9) for $Q_{\rm e}$ and $Q_{\rm m}$, we find the dyon mass and the relation between angles α and γ:

$$m_{\rm dyon} = a\frac{4\pi c}{e} = gvc^2, \qquad \cos\alpha = \frac{1}{c}, \qquad \sin\alpha = -\frac{s}{c}, \tag{15}$$

where we have introduced abbreviations $c \equiv \cosh\gamma$ and $s \equiv \sinh\gamma$. The Bogomol'nyi equations (14) are indeed satisfied by the following properties of functions K and H:

$$xK' = -KH, \qquad xH' = H-(K^2-1). \tag{16}$$

It should be noted that the BPS state considered here preserves half of the supersymmetry, as is generally the case for BPS solutions. To show this explicitly, we use the following explicit representation for the γ-matrices:

$$\gamma^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = 1\otimes\sigma_2, \qquad \boldsymbol{\gamma} = \begin{pmatrix} -i\boldsymbol{\sigma} & 0 \\ 0 & i\boldsymbol{\sigma} \end{pmatrix} = \boldsymbol{\sigma}\otimes(-i\sigma_3). \tag{17}$$

Then, using the dyon solution (4) and the relations

$$\boldsymbol{D}S^a = c\boldsymbol{B}^a, \qquad \boldsymbol{E}^a = -s\boldsymbol{B}^a \tag{18}$$

which follow from the Bogomol'nyi equations (14), the supersymmetry transformation $\delta\psi$ of the fermion in (3) takes the form

$$\delta\psi^a = \begin{pmatrix} i(\boldsymbol{B}^a+\boldsymbol{D}S^a)\cdot\boldsymbol{\sigma} & \boldsymbol{E}^a\cdot\boldsymbol{\sigma} \\ \boldsymbol{E}^a\cdot\boldsymbol{\sigma} & i(\boldsymbol{B}^a-\boldsymbol{D}S^a)\cdot\boldsymbol{\sigma} \end{pmatrix}\tilde{\alpha} = 2i\boldsymbol{B}^a\cdot\boldsymbol{\sigma}\otimes P_+(\gamma)\tilde{\alpha}, \tag{19}$$

where $P_+(\gamma)$ is a projection operator. It and the other projection operator, $P_-(\gamma)$, are given by

$$P_\pm(\gamma) \equiv \frac{1}{2}\begin{pmatrix} 1 \pm c & \pm is \\ \pm is & 1 \mp c \end{pmatrix} = \frac{1 \pm (c\sigma_3 + is\sigma_1)}{2} = e^{\frac{\gamma}{2}\sigma_2}\left(\frac{1 \pm \sigma_3}{2}\right)e^{-\frac{\gamma}{2}\sigma_2}. \tag{20}$$

From Eq. (19), it is seen that only half of the supersymmetry parameter, constituted by $P_+(\gamma)\tilde{\alpha}$, is involved in $\delta\psi$, while the remaining half, $P_-(\gamma)\tilde{\alpha}$, corresponds to the unbroken supersymmetry.

Now, let us perform a finite supersymmetry transformation of the Julia-Zee dyon solution with the following form of the transformation parameter $\tilde{\alpha}$, parametrized by a two-component Grassmann parameter α:

$$\tilde{\alpha} \equiv P_+(\gamma)\begin{pmatrix} \alpha \\ 0 \end{pmatrix} = \frac{1}{2}\,\alpha \otimes \begin{pmatrix} 1 + c \\ is \end{pmatrix}. \tag{21}$$

Any field $\varPhi$ is transformed under a finite supersymmetry transformation as

$$\begin{aligned} \tilde{\varPhi} &= e^{i(\bar{\tilde{\alpha}}Q + \bar{Q}\tilde{\alpha})}\varPhi e^{-i(\bar{\tilde{\alpha}}Q + \bar{Q}\tilde{\alpha})} \\ &= \varPhi + \delta\varPhi + \frac{1}{2}\delta^2\varPhi + \frac{1}{3!}\delta^3\varPhi + \frac{1}{4!}\delta^4\varPhi. \end{aligned} \tag{22}$$

This series terminates at the fourth-order term in α since it is a two-component complex Grassmann parameter. In the computations involved in the present transformation of the classical solution, it is easier to calculate $\delta^n\varPhi$ in the form $\delta^{n-1}(\delta\varPhi)$ than $\delta(\delta^{n-1}\varPhi)$, because we can substitute the classical solution only at the end of the calculation, therefore we can use the result of the preceding step, $\delta^{n-1}\varPhi$, only in the final step.

Any solution of the equation of motion is transformed to another solution under a finite symmetry transformation, since the action is invariant under such a transformation. Therefore, the image of the Julia-Zee dyon solution under a finite supersymmetry transformation is also an exact solution of the equation of motion for any choice of the transformation parameter α. The first-order term in α of $\tilde{\varPhi}(\alpha)$ gives the massless fermion solution around the Julia-Zee dyon background. The higher-order terms describe the back-reaction of the created fermion. The computation of these terms is carried out in Refs. 2) and 3) for the case of the monopole solution (i.e., the $\theta = 0$ case). In those works, it is found that the gyroelectric ratio, g_{e}, is just equal to the Dirac value, 2, for the monopole fermion.

We now perform the same calculation for the Julia-Zee dyon case and, in particular, examine the electric dipole moment of the Julia-Zee dyon fermion to determine whether $g_{\mathrm{e}} = 2$ holds also in the $\theta \neq 0$ case.

At first-order in α, only the fermion, ψ, is nonvanishing; all the boson fields are zero because the fermion field ψ vanishes at zeroth order:

$$\delta^1\psi^a = i\boldsymbol{B}^a\cdot\boldsymbol{\sigma}\alpha\otimes\begin{pmatrix}1+c\\ is\end{pmatrix}, \qquad \delta^1 A^a_\mu = \delta^1 S^a = \delta^1 P^a = 0. \tag{23}$$

Conversely, the fermion vanishes at second order, since all the bosons vanish at the first order:

$$\begin{pmatrix}\delta^2 A^a_0\\ \delta^2 S^a\\ \delta^2 P^a\end{pmatrix} = 2(1+c)(\alpha^\dagger\boldsymbol{B}^a\cdot\boldsymbol{\sigma}\alpha)\times\begin{pmatrix}-c\\ -s\\ 1\end{pmatrix}, \qquad \delta^2\boldsymbol{A}^a = 0, \quad \delta^2\psi^a = 0. \tag{24}$$

Note that $\delta^2\boldsymbol{A}^a$ vanishes because we are restricting the transformation parameter to that part projected by $P_+(\gamma)$. Then, at third order, only the fermion could be non-vanishing, but with a straightforward calculation using the relations among $\delta^2 A^a_0$, $\delta^2 S^a$ and $\delta^2 P^a$ given in (24) and the relation $A^a_0 = (s/c)S^a$, we can show that it actually vanishes. Explicitly, we have

$$\begin{aligned}\delta^3\psi &= \delta^2\left(\frac{1}{2}\gamma^{\mu\nu}F_{\mu\nu} - \gamma^\mu D_\mu S + i\gamma^\mu D_\mu P\gamma_5 - e[P,S]\gamma_5\right)P_+(\gamma)\tilde{\alpha}\\ &\propto (\alpha^\dagger\sigma^i\alpha)\Big(\big(\boldsymbol{\sigma}\cdot\boldsymbol{D}B^i + \frac{e}{c}[S,\, B^i]\big)\otimes(c\sigma_1 - is\sigma_3 + i\sigma_2)\Big)P_+(\gamma)\tilde{\alpha}\,.\end{aligned} \tag{25}$$

Noting the relation $c\sigma_1 - is\sigma_3 + i\sigma_2 = i\sigma_2 P_-(\gamma)$, this clearly vanishes, due to the orthogonality $P_-(\gamma)P_+(\gamma) = 0$. Thus, all the fields are zero at third order, and hence also at fourth order.

Now that we have obtained the finite SUSY transformation of the Julia-Zee dyon solution, we are able to derive the following form for the $U(1)$ electric field:

$$\begin{aligned}\tilde{\boldsymbol{E}} = \hat{r}^a\tilde{\boldsymbol{E}}^a &= -\hat{r}^a\boldsymbol{D}\tilde{A}^a_0 = -\hat{r}^a\boldsymbol{D}(A^a_0 + \frac{1}{2}\delta^2 A^a_0)\\ &= -s\hat{r}^a\boldsymbol{B}^a + c(1+c)\hat{r}^a\boldsymbol{D}(\alpha^\dagger\boldsymbol{B}^a\cdot\boldsymbol{\sigma}\alpha).\end{aligned} \tag{26}$$

Then, inserting the expression for the magnetic field

$$B^{ia} = \frac{\hat{r}^i\hat{r}^a}{er^2}(1-K^2) + \frac{\delta^{ia}_\perp}{er^2}KH\ , \tag{27}$$

with the transverse Kronecker delta $\delta^{ij}_\perp \equiv \delta^{ij} - \hat{r}^i\hat{r}^j$, we obtain

$$\tilde{E}^i = -\frac{s\hat{r}^i}{er^2}(1-K^2) + c(1+c)\frac{3\hat{r}^i\hat{r}^j - \delta^{ij}}{er^3}(K^2H + K^2 - 1)(\alpha^\dagger\sigma^j\alpha)\ . \tag{28}$$

The first term here is merely the Coulomb field around the dyon charge $Q_{\rm e} = -gs$, since we have $1/e = g/4\pi$. The second term is the electric dipole field induced around the source

fermion field, $\delta^1\psi$ (23).*) Next, noting the asymptotic behavior $H(x)/x \to 1$, $K(x) \to 0$ from this term, we find that the dyon fermion around the origin possesses the electric dipole moment

$$\boldsymbol{\mu}_{\mathrm{e}} = -\frac{4\pi c(1+c)}{e}(\alpha^\dagger \boldsymbol{\sigma} \alpha)\,. \tag{29}$$

In order to relate the operator $(\alpha^\dagger \boldsymbol{\sigma}\alpha)$ here to the spin operator $\boldsymbol{J}$, we must determine the normalization of our two-component spinor α. For this purpose, we calculate the fermion number carried by our dyon fermion state $\delta^1\psi$. This is done as follows:

$$\begin{aligned} 1 &= \int d^3x\, (\delta^1\psi^a)^\dagger(\delta^1\psi^a) = 2c(1+c)\int d^3x\, B^{ia}B^{aj}(\alpha^\dagger\sigma^i\sigma^j\alpha) \\ &= 2c(1+c)\frac{m_{\mathrm{dyon}}}{c^2}(\alpha^\dagger\alpha)\ . \end{aligned} \tag{30}$$

Here we have used $\int d^3x\, \boldsymbol{B}^a\cdot\boldsymbol{B}^a = m_{\mathrm{dyon}}/c^2$, which follows from (12) and (18). The 'number operator' should be given by $a^\dagger a$, where $a^\dagger$ and a are the properly normalized (two-component spinor) creation and annihilation operators, in terms of which the spin operator $\boldsymbol{J}$ reads $a^\dagger(\boldsymbol{\sigma}/2)a$. Identifying the right-hand side of (30) with the number operator, we find

$$2c(1+c)\frac{m_{\mathrm{dyon}}}{c^2}\left(\alpha^\dagger\frac{\boldsymbol{\sigma}}{2}\alpha\right) = \boldsymbol{J}\,. \tag{31}$$

In terms of this spin, the electric dipole moment of the dyon fermion is given by

$$\boldsymbol{\mu}_{\mathrm{e}} = -\frac{4\pi c^2}{e m_{\mathrm{dyon}}}\boldsymbol{J} = -2c^2\frac{g}{2m_{\mathrm{dyon}}}\boldsymbol{J}\,, \tag{32}$$

where the unit magnetic charge $4\pi/e$ is denoted by g. This result implies that the gyroelectric ratio g_{e} of the dyon fermion is $2c^2$.

Recall that the parameter γ is quantized as in (11). Using this, we can rewrite $c^2 = \cosh^2\gamma$ in terms of the electric quantum number n_{e} and the θ-parameter. Doing so, we find that the gyroelectric ratio of the dyon fermion with quantum numbers $n_{\mathrm{m}} = 1$ and n_{e} is given by

$$g_{\mathrm{e}} = 2\left(1 + \left(\frac{e^2\theta}{8\pi^2} + n_{\mathrm{e}}\frac{e^2}{4\pi}\right)^2\right). \tag{33}$$

It is interesting that even the monopole fermion with $n_{\mathrm{m}} = 1$, $n_{\mathrm{e}} = 0$ has a gyroelectric ratio that deviates from the canonical Dirac value of 2 in the $\theta \neq 0$ case.

It is also interesting that there are no $O(\alpha^2)$ back-reactions to the magnetic field, as seen from the relation $\delta^2\boldsymbol{A}^a = 0$ in (24). For this reason, there is no dipole part in the magnetic

*) Actually, the sources of this electric dipole field include the dipole moment of the gauge boson charge distribution and the EDM due to magnetic current, in addition to the dipole moment of the fermion charge distribution. These contributions were calculated separately in a previous paper[3)] by one of the present authors.

field around the dyon,

$$\boldsymbol{B} \equiv \hat{r}^a \boldsymbol{B}^a = \frac{\hat{\boldsymbol{r}}}{er^2}(1-K^2)\,, \tag{34}$$

and thus the asymptotic behavior is simply given by that of a magnetic monopole. For later convenience, we here write the asymptotic forms of the electric and magnetic fields around the monopole fermion (with $n_{\rm m}=1$, $n_{\rm e}=0$):

$$\begin{cases} \boldsymbol{E} = \dfrac{e\theta/2\pi}{4\pi}\dfrac{\hat{\boldsymbol{r}}}{r^2} + \dfrac{4\pi/e}{m}c^2\boldsymbol{D}_{\rm dipole} \\ \boldsymbol{B} = \dfrac{4\pi/e}{4\pi}\dfrac{\hat{\boldsymbol{r}}}{r^2} + 0\boldsymbol{D}_{\rm dipole} \end{cases} \qquad \begin{array}{l}\text{as} \quad r\to\infty \quad \text{around} \\ \text{a monopole fermion}\end{array} \quad . \tag{35}$$

Here $\boldsymbol{D}_{\rm dipole}$ is the dipole field, defined as

$$\boldsymbol{D}_{\rm dipole} \equiv \frac{1}{4\pi}\frac{3\hat{\boldsymbol{r}}\,(\hat{\boldsymbol{r}}\cdot\boldsymbol{J}) - \boldsymbol{J}}{r^3} \tag{36}$$

in terms of the spin operator $\boldsymbol{J}$.

§3. $SL(2;\mathbb{Z})$ duality transformation of the field strength

We have calculated the electric and magnetic fields around a dyon fermion and found the corresponding values of the electric and magnetic dipole moments. We next set out to obtain the dipole moments in the dual world from these results. For this purpose, we need the $SL(2;\mathbb{Z})$ duality transformation of the gauge field strength. The duality transformation of electromagnetic field has been known for a long time as given, e.g., in Refs. 8), 9). Here, however, we recapitulate it in our notation in this section for the explicit use below.

First, recall the $SL(2;\mathbb{Z})$ duality argument of Seiberg and Witten. In that argument, the original $SU(2)$ supersymmetric Yang-Mills system is described by an effective $U(1)$ supersymmetric gauge theory in the low energy region. Then, with the rescaling $eA_\mu \to A_\mu$, the action $S_{\rm o}$ of the $U(1)$ gauge field part of the effective theory takes the form

$$\begin{aligned} -S_{\rm o} &= \frac{1}{32\pi}{\rm Im}\int \tau\,(F-i\tilde{F})^2 = \frac{1}{16\pi}{\rm Im}\int \tau\,(F^2 - i\tilde{F}\cdot F) \\ &= \int\Big(\frac{1}{4e^2}F^2 - \frac{\theta}{32\pi^2}\tilde{F}\cdot F\Big), \end{aligned} \tag{37}$$

where τ is defined as

$$\tau \equiv \frac{\theta}{2\pi} + i\frac{4\pi}{e^2}\ . \tag{38}$$

Note that the form given in (37) follows from the relations $\tilde{\tilde{F}} = -F$ and $F\cdot\tilde{G} = \tilde{F}\cdot G$ for the dual field strength $\tilde{F}^{\mu\nu} \equiv (1/2)\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$. Now, ignoring the definition $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$,

we regard $F_{\mu\nu}$ as an independent variable that is constrained by the Bianchi identity; then we have

$$
\begin{aligned}
-S &= -S_{\rm o} + \frac{1}{8\pi}\int V_{{\rm D}\mu}\epsilon^{\mu\nu\rho\sigma}\partial_\nu F_{\rho\sigma} = -S_{\rm o} + \frac{1}{8\pi}\int \tilde{F}_{\rm D}\cdot F \\
&= -S_{\rm o} - \frac{1}{16\pi}{\rm Im}\int (F_{\rm D} - i\tilde{F}_{\rm D})(F - i\tilde{F}) \ , \qquad (39)
\end{aligned}
$$

with the dual system field strength $F_{{\rm D}\mu\nu} \equiv \partial_\mu V_{{\rm D}\nu} - \partial_\nu V_{{\rm D}\mu}$. We can now dual transform this system as follows. First, completing the square as

$$
\begin{aligned}
-S &= \frac{1}{32\pi}{\rm Im}\int \left[\tau(F - i\tilde{F})^2 - 2(F_{\rm D} - i\tilde{F}_{\rm D})(F - i\tilde{F})\right] \\
&= \frac{1}{32\pi}{\rm Im}\int \left[\tau\left(F - i\tilde{F} - \frac{1}{\tau}(F_{\rm D} - i\tilde{F}_{\rm D})\right)^2 - \frac{1}{\tau}(F_{\rm D} - i\tilde{F}_{\rm D})^2\right] , \qquad (40)
\end{aligned}
$$

we perform the Gaussian integration over F. The same result can be obtained by employing the equation of motion

$$
{\rm Im}\left(\tau(F - i\tilde{F}) - (F_{\rm D} - i\tilde{F}_{\rm D})\right) = 0 \ , \qquad (41)
$$

which implies that the real part also vanishes so that

$$
\tau(F - i\tilde{F}) = F_{\rm D} - i\tilde{F}_{\rm D} \, . \qquad (42)
$$

Thus we obtain the dual action

$$
-S_{\rm D} = \frac{1}{32\pi}{\rm Im}\int\left(-\frac{1}{\tau}\right)(F_{\rm D} - i\tilde{F}_{\rm D})^2 \, . \qquad (43)
$$

From (42), we find

$$
F_{\rm D} = {\rm Re}\left(\tau(F - i\tilde{F})\right) = {\rm Re}\tau\, F + {\rm Im}\tau\, \tilde{F} \, . \qquad (44)
$$

This implies that for the electromagnetic fields

$$
\begin{cases}
e\boldsymbol{E} = -(F^{01}\, F^{02},\, F^{03}), \\
e\boldsymbol{B} = -(F^{23}\, F^{31},\, F^{12}),
\end{cases}
\qquad
\begin{cases}
e_{\rm D}\boldsymbol{E}_{\rm D} = -(F^{01}_{\rm D}\, F^{02}_{\rm D},\, F^{03}_{\rm D}), \\
e_{\rm D}\boldsymbol{B}_{\rm D} = -(F^{23}_{\rm D}\, F^{31}_{\rm D},\, F^{12}_{\rm D}),
\end{cases}
\qquad (45)
$$

we have the following mapping:

$$
\begin{cases}
e_{\rm D}\boldsymbol{E}_{\rm D} = \dfrac{4\pi}{e}\boldsymbol{B} + \dfrac{e\theta}{2\pi}\boldsymbol{E} \\
e_{\rm D}\boldsymbol{B}_{\rm D} = -\dfrac{4\pi}{e}\boldsymbol{E} + \dfrac{e\theta}{2\pi}\boldsymbol{B}
\end{cases} . \qquad (46)
$$

Note that from the definition $\tau_{\rm D} \equiv -1/\tau$, we have

$$
\begin{aligned}
e_{\rm D} &= \sqrt{\left(\frac{4\pi}{e}\right)^2 + \left(\frac{e\theta}{2\pi}\right)^2} = \frac{4\pi}{e}c \ , \\
e_{\rm D}^2\theta_{\rm D} &= -e^2\theta \ , \qquad (47)
\end{aligned}
$$

with

$$\begin{cases} s \equiv -\dfrac{e^2\theta}{8\pi^2} \\ c = \sqrt{1+s^2} = \sqrt{1+\left(\dfrac{e^2\theta}{8\pi^2}\right)^2} \end{cases} \quad . \tag{48}$$

Here, c and s represent the quantities $\cosh\gamma$ and $\sinh\gamma$ introduced previously for the dyon solutions. Also note that $c = e\,e_{\rm D}/4\pi$ is a duality invariant quantity, as is s, up to its sign.

To this point, we have considered only the genuine duality transformation, usually called the S-transformation:

$$S: \quad \tau \;\to\; -\frac{1}{\tau}\ . \tag{49}$$

Now, let us consider the generalized duality transformation $SL(2;\mathbb{Z})$, which is generated by this S-transformation and the T-transformation

$$T: \quad \tau \;\to\; \tau + 1\ , \tag{50}$$

corresponding to a shift by 2π of the θ parameter. The transformation is generally given by

$$\tau \;\to\; \tau' = \frac{a\tau + b}{c\tau + d} \tag{51}$$

for 2×2 matrices $M \in SL(2;\mathbb{Z})$

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\ , \quad \text{where} \quad a,b,c,d \in \mathbb{Z}, \quad ad - bc = 1\ . \tag{52}$$

Following Ref. 8), we identify the following quantity of the field strength

$$\begin{pmatrix} F_{\rm D} - i\tilde{F}_{\rm D} \equiv \tau\,(F - i\tilde{F}) \\ F - i\tilde{F} \end{pmatrix}\ . \tag{53}$$

as an $SL(2;\mathbb{Z})$ vector. Then, the gauge field generally transforms under $SL(2;\mathbb{Z})$ as

$$\begin{pmatrix} F_{\rm D} - i\tilde{F}_{\rm D} \\ F - i\tilde{F} \end{pmatrix} \;\to\; \begin{pmatrix} F'_{\rm D} - i\tilde{F}'_{\rm D} \\ F' - i\tilde{F}' \end{pmatrix} = M \begin{pmatrix} F_{\rm D} - i\tilde{F}_{\rm D} \\ F - i\tilde{F} \end{pmatrix}\ . \tag{54}$$

Indeed, the relation $F_{\rm D} - i\tilde{F}_{\rm D} = \tau\,(F - i\tilde{F})$ still holds after the transformation; that is, we have $F'_{\rm D} - i\tilde{F}'_{\rm D} = \tau'(F' - i\tilde{F}')$, which is consistent with the S transformation discussed above,

$$S: \begin{pmatrix} F_{\rm D} - i\tilde{F}_{\rm D} \\ F - i\tilde{F} \end{pmatrix} \;\to\; \begin{pmatrix} -(F - i\tilde{F}) \\ F_{\rm D} - i\tilde{F}_{\rm D} \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} F_{\rm D} - i\tilde{F}_{\rm D} \\ F - i\tilde{F} \end{pmatrix} \tag{55}$$

where use has been made of the relation $F'_D - i\tilde{F}'_D = \tau'(F' - i\tilde{F}') = (-1/\tau)(F_D - i\tilde{F}_D) = -(F - i\tilde{F})$. Next, note that from the definition (53), we immediately obtain the following transformation rule under T:

$$T: \begin{pmatrix} F_{\mathrm{D}} - i\tilde{F}_{\mathrm{D}} \\ F - i\tilde{F} \end{pmatrix} \rightarrow \begin{pmatrix} (F - i\tilde{F}) + (F_{\mathrm{D}} - i\tilde{F}_{\mathrm{D}}) \\ (F - i\tilde{F}) \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} F_{\mathrm{D}} - i\tilde{F}_{\mathrm{D}} \\ F - i\tilde{F} \end{pmatrix} . \tag{56}$$

Transformation rule given in (54) takes the same form as that of Seiberg and Witten for the complex scalar fields:

$$\begin{pmatrix} a_{\mathrm{D}} \\ a \end{pmatrix} \rightarrow M \begin{pmatrix} a_{\mathrm{D}} \\ a \end{pmatrix} . \tag{57}$$

Here, a stands for the VEV of the complex scalar field $a = (S^3 + iP^3)$, and a_{D} can be identified with τa similarly to Eq. (53).

In order to express the mass formula in terms of the quantum numbers n_{e} and n_{m}, let us write the charges Q_{e} in (11) and $Q_{\mathrm{m}} = n_{\mathrm{m}}(4\pi/e)$ in the form

$$Q_{\mathrm{e}} + iQ_{\mathrm{m}} = e(n_{\mathrm{e}} + n_{\mathrm{m}}\tau) \ . \tag{58}$$

Then the mass formula for the BPS states that saturate the bound can be written

$$m = a\sqrt{Q_{\mathrm{e}}^2 + Q_{\mathrm{m}}^2} = ae\,|n_{\mathrm{m}}\tau + n_{\mathrm{e}}| = e\,|Z| \ \ , \tag{59}$$

where

$$Z = n_{\mathrm{m}} a_{\mathrm{D}} + n_{\mathrm{e}} a = (n_{\mathrm{m}},\ n_{\mathrm{e}}) \begin{pmatrix} a_{\mathrm{D}} \\ a \end{pmatrix} . \tag{60}$$

In order for Z to remain intact under $SL(2;\mathbb{Z})$, the quantum numbers should transform as

$$(n_{\mathrm{m}},\, n_{\mathrm{e}}) \ \rightarrow \ (n_{\mathrm{m}},\, n_{\mathrm{e}})M^{-1} \,. \tag{61}$$

§4. *S*-duality transformation of a monopole fermion to an electron

Now we apply the S duality transformation given in (55) to the monopole fermion state with quantum numbers $(n_{\mathrm{m}},\, n_{\mathrm{e}}) = (1,\, 0)$. Because the quantum number is transformed to $(1,\, 0)S^{-1} = (0,\, 1)$ by the transformation appearing in (61), the monopole fermion state is transformed to the electron state with $(n_{\mathrm{m}},\, n_{\mathrm{e}}) = (0,\, 1)$. Therefore, we can find the asymptotic electromagnetic field around the electron by applying the S-duality transformation (46) to the asymptotic electromagnetic field (35) around the monopole fermion. First, we compute the Coulomb terms alone, deferring the dipole terms to the next step:

$$\begin{cases} \boldsymbol{E}_{\mathrm{D}}\Big|_{\mathrm{Coulomb}} = \left(\dfrac{4\pi}{ee_{\mathrm{D}}}\dfrac{1}{e} + \dfrac{e\theta}{2\pi e_{\mathrm{D}}}\dfrac{e\theta/2\pi}{4\pi}\right) \times \dfrac{\hat{\boldsymbol{r}}}{r^2} = \dfrac{e_{\mathrm{D}}}{4\pi}\dfrac{\hat{\boldsymbol{r}}}{r^2} \\ \boldsymbol{B}_{\mathrm{D}}\Big|_{\mathrm{Coulomb}} = \left(-\dfrac{4\pi}{ee_{\mathrm{D}}}\dfrac{e\theta/2\pi}{4\pi} + \dfrac{e\theta}{2\pi e_{\mathrm{D}}}\dfrac{1}{e}\right) \times \dfrac{\hat{\boldsymbol{r}}}{r^2} = 0\dfrac{\hat{\boldsymbol{r}}}{r^2} \end{cases} . \tag{62}$$

Here, we have used the expression (47) for the charge e_{D} in the dual world. This Coulomb part merely reconfirms that the quantum numbers change as $(1,0) \rightarrow (0,1)$ from the monopole fermion state to electron state under the S-transformation. Next, we calculate the dipole field part, which comes only from $\boldsymbol{E}$:

$$\begin{cases} \boldsymbol{E}_{\mathrm{D}}\Big|_{\mathrm{dipole}} = \dfrac{e\theta}{2\pi e_{\mathrm{D}}}\dfrac{4\pi/e}{m}c^2\boldsymbol{D}_{\mathrm{dipole}} = \dfrac{e^2\theta}{8\pi^2}\dfrac{e_{\mathrm{D}}}{m}\boldsymbol{D}_{\mathrm{dipole}} \\ \boldsymbol{B}_{\mathrm{D}}\Big|_{\mathrm{dipole}} = -\dfrac{4\pi}{ee_{\mathrm{D}}}\dfrac{4\pi/e}{m}c^2\boldsymbol{D}_{\mathrm{dipole}} = -\dfrac{e_{\mathrm{D}}}{m}\boldsymbol{D}_{\mathrm{dipole}} \end{cases} . \tag{63}$$

This result represents the values of the electric and magnetic dipole moments of the electron in the dual world. We should note that this dual electron world is *different* from the original monopole world; indeed, if the monopole world is a strong coupling world, i.e. $e \gg 1$, then the electron world is a weak coupling world, i.e. $e_{\mathrm{D}} \ll 1$, and the VEV of the scalar field is $a = cv$ in the monopole world while it is $a' = a_{\mathrm{D}} = \tau a$ in the electron world. Thus the electron mass in the dual world is given by

$$m_{\mathrm{e}} = e\left|0 \times a'_{\mathrm{D}} + 1 \times a'\right| = e\left|a_{\mathrm{D}}\right| = m\ , \tag{64}$$

which is, as should be the case, the same as the monopole dyon mass m in the original world. Therefore, the result for $\boldsymbol{B}_{\mathrm{D}}\big|_{\mathrm{dipole}}$ given in (63) shows that the magnetic moment of this electron is exactly Dirac's value of $2 \times (e_{\mathrm{D}}/2m)J$. It is interesting that the magnetic moment of the electron possesses this canonical value for any coupling strength of this electron world. This is probably due to the $N = 2$ supersymmetry of the system which forbids higher-order radiative corrections to the magnetic moment.

We are now ready to state the most important result of this paper. From the electric dipole field $\boldsymbol{E}_{\mathrm{D}}\big|_{\mathrm{dipole}}$ given in (63), we find that the electron in this dual world carries the EDM

$$\frac{e^2\theta}{8\pi^2}\frac{e_{\mathrm{D}}}{m}J = -\frac{e_{\mathrm{D}}^2\theta_{\mathrm{D}}}{8\pi^2}\frac{e_{\mathrm{D}}}{m}J\ . \tag{65}$$

It is thus seen that the EDM is equal to $-(e_{\mathrm{D}}^2\theta_{\mathrm{D}}/8\pi^2)$ times the Dirac magnetic moment.

We regard this dual world as a (toy) model of our world and the charged, spin 1/2 fermion treated here as corresponding to electron. Note that the multiplet consisting of this fermion and a scalar is a BPS state, as is the original monopole dyon multiplet, and it constitutes a small supermultiplet consisting of spin 1/2 and 0, on which half of the supercharges vanish. Because e_{D} and θ_{D} are the coupling constant and the θ parameter in *our world*, we rewrite our final result for the EDM of the electron by dropping the subscript "D", which indicates the dual world, and using $\alpha \equiv e^2/4\pi$:

$$\mu_{\mathrm{e}} = -\frac{e^2\theta}{8\pi^2}\frac{e}{m}J = -\alpha\frac{\theta}{2\pi}\frac{e}{m}J\ . \tag{66}$$

§5. Reflection and discussion

Considering the result (66) for the electron EDM, we find a strange property: This quantity is not invariant under the shift θ, $\theta \rightarrow \theta + 2\pi$. Because the system should be 2π-periodic in θ, something might be wrong.

First, note that Witten's induced charge, $e\theta/2\pi$, for the monopole is obviously not periodic in θ. However, we can always perform the T-transformation $\tau \rightarrow \tau + 1$, which shifts θ by 2π and simultaneously transforms the monopole dyon charge (n_m=1, n_e) to (n_m=1, $n_\mathrm{e} - 1$). Thus, if we shift θ by 2π and apply the T-transformation, the electric charge $Q_\mathrm{e} = n_\mathrm{e} e + e\theta/2\pi$ of the state is unchanged. Therefore, the periodicity of the system implies that all the states in the tower of the monopole dyons with n_m=1 and $n_\mathrm{e} \in \mathbb{Z}$ must exist in the system. For a given θ, any one of the states in this tower of monopole dyons can be transformed to the electron state by applying an $SL(2;\mathbb{Z})$ transformation. To illustrate this, let us suppose that the initial state has the quantum numbers n_m=1 and n_e. We first perform the T-transformation n_e times, reducing the electric quantum number n_e to zero. Then θ is changed to $\theta + 2n_\mathrm{e}\pi$. The resulting state possesses the monopole's quantum numbers, i.e. $(n_\mathrm{m}, n_\mathrm{e})$ = (1, 0). Then, applying the S-transformation, this state is transformed to the electron state, with $(n_\mathrm{m}, n_\mathrm{e})$ = (0, 1). It is thus seen that applying the transformation ST^{n_e} to the starting state (n_m=1, n_e), we can obtain the electron state, which carries the EDM

$$\frac{e^2(\theta + 2n_\mathrm{e}\pi)}{8\pi^2}\frac{e_\mathrm{D}}{m}J = -\frac{e_\mathrm{D}^2\theta_\mathrm{D}}{8\pi^2}\frac{e_\mathrm{D}}{m}J \ , \tag{67}$$

where the coupling constant e_D and the θ_D parameter are given in terms of the original parameters e and θ by

$$\begin{aligned} e_\mathrm{D} &= e\sqrt{\left(\frac{4\pi}{e^2}\right)^2 + \left(\frac{\theta}{2\pi} + n_\mathrm{e}\right)^2} = e\,|n_\mathrm{e} + n_\mathrm{m}\tau| \ , \\ \theta_\mathrm{D} &= -\frac{\theta + 2n_\mathrm{e}\pi}{|n_\mathrm{e} + \tau|^2} = -2\pi \mathrm{Re}\left(\frac{1}{n_\mathrm{e} + n_\mathrm{m}\tau}\right) , \end{aligned} \tag{68}$$

with $n_\mathrm{m} = 1$. Here the result depends on the electric quantum number n_e of the starting state in the dyon tower. This result thus shows that the invariance under the shift $\theta \rightarrow \theta + 2\pi$ can be realized by shifting n_e of the starting state in the tower. More importantly, the EDM of the electron state is given uniquely by the expression (66) if it is written in terms of the the coupling constant e and the θ parameter in the electron world, in which the electron exists. Despite this invariance, we think that the starting state should be that with the minimum mass among those states in this dyon fermion tower that satisfy the condition $-1/2 < (\theta/2\pi) + n_\mathrm{e} \leq 1/2$, so that the electron has the smallest possible mass.

Despite the above considerations, the fact remains that the result given in (66) is not invariant under a 2π shift of θ in the electron world. However, we claim that it is not necessary for the result to be 2π-periodic in θ after the S-duality transformation. Our argument for this claim is as follows. First of all, the θ-term in the $U(1)$ gauge theory after the S-duality transformation is not directly related to the θ-term of a non-Abelian gauge theory, which has a topological meaning. The θ-term in a mere $U(1)$ gauge theory is trivial, because $\pi_3(U(1)) = 0$, and hence no periodicity in θ is necessary. In the present electron world considered presently, the θ-term, which is proportional to $\theta \boldsymbol{E} \cdot \boldsymbol{B}$, has the following meaning that it causes magnetically charged objects to possess the Witten charge. Indeed, this system has a BPS monopole multiplet with $n_{\mathrm{m}}= \pm 1, n_{\mathrm{e}}=0$, which is obtained by the S-duality transformation of the W-boson multiplet. Thus measuring the electric charge $Q_{\mathrm{e}} = e\theta/2\pi$ of this monopole multiplet, we should be able to determine the value of θ.*)

We called the fermion dual to the monopole fermion, 'electron'. However, if we seriously want to identify this object with the real electron, there are many problems. First of all, the present computation heavily depends on the existence of $N = 2$ supersymmetry. We do not know how much our computation can be generalized to the systems with $N = 1$ or no supersymmetry. Moreover, it is completely unclear at the moment how the usual Standard Model with quarks and leptons can be reconstructed or embedded in such a dual world in which our real electron is identified with an object dual to the monopole fermion in a certain non-Abelian gauge theory.

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*) As we claim that there is no necessity for the periodicity of μ_{e} in θ, we also believe that there exists no T-transformation tower states of the monopole, with quantum numbers $n_{\mathrm{m}}= \pm 1$, $n_{\mathrm{e}} \in \mathbb{Z}$. We thus believe that the electric charge Q_{e} of the monopole multiplet is given solely by the Witten effect.

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